**RK COMBINE**

**#RK Methods (ALL COMBINE---> Euler, Heun, Midpoint, Raltson, 3rd, 4th, 5th order)**

**import matplotlib.pyplot as plt**

**[xo,xn,yo\_e,yo\_h,yo\_m,yo\_r,yo\_3,yo\_4,yo\_5,yo\_t,h]=[0,4,1,1,1,1,1,1,1,1,0.5]**

**n=(xn-xo)/h**

**#f(x.y)=dy/dx (i.e derivative/slope of original; function)**

**print("COMBINE METHOD EXAMPLE")**

**def f1(x,y):**

**return -2\*(x)\*\*3 + 12\*(x)\*\*2 - 20\*(x) + 8.5**

**#f(x,y)=y(i.e integral of f1 which is original function)**

**def f2(x,y):**

**return -0.5\*(x)\*\*4 + 4\*(x)\*\*3 - 10\*(x)\*\*2 + 8.5\*(x) + 1**

**print("xi y-euler y-heun y-mid y-ral y-3rd y-4th y-5th y-true")**

**print("===========================================================================")**

**A,B,C,D,E,F,G,H,I=[],[],[],[],[],[],[],[],[]**

**for i in range(0,int(n)+1):**

**yi\_euler=yo\_e+(f1(xo,yo\_e))\*h**

**yi\_heun=yo\_h+((1/2)\*(f1(xo,yo\_h)+(f1(xo+h,yo\_h+(h\*f1(xo,yo\_h))))))\*h**

**yi\_mid=yo\_m+(f1(xo+h/2,yo\_m+(1/2)\*(f1(xo,yo\_m))\*h))\*h**

**yi\_ral=yo\_r+(((1/3)\*(f1(xo,yo\_r)))+((2/3)\*(f1(xo+(3/4)\*h,yo\_r+(3/4)\*(f1(xo,yo\_r))\*h))))\*h**

**yi\_rk3=yo\_3+((1/6)\*((f1(xo,yo\_3))+4\*(f1(xo+h/2,yo\_3+(f1(xo,yo\_3)\*h/2)))+f1(xo+h,yo\_3-h\*(f1(xo,yo\_3))+2\*h\*((f1(xo+h/2,yo\_3+(f1(xo,yo\_3)\*h/2))))))\*h)**

**yi\_rk4=yo\_4+((1/6)\*(f1(xo,yo\_4)+2\*(f1(xo+h/2,yo\_4+(f1(xo,yo\_4)\*h/2)))+2\*(f1(xo+h/2,yo\_4+(h/2)\*(f1(xo+h/2,yo\_4+(h/2)\*(f1(xo,yo\_4))))))+f1(xo+h,yo\_4+h\*(f1(xo+h/2,yo\_4+(h/2)\*(f1(xo+h/2,yo\_4+(h/2)\*(f1(xo,yo\_4)))))))))\*h**

**yi\_rk5=yo\_5+((1/90)\*(7\*(f1(xo,yo\_5))+32\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))+12\*(f1(xo+h/2,yo\_5-(h/2)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))+h\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))))+32\*(f1(xo+(3/4)\*h,yo\_5+(3/16)\*h\*(f1(xo,yo\_5))+(9/16)\*h\*(f1(xo+h/2,yo\_5-(h/2)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))+h\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))))))+7\*(f1(xo+h,yo\_5-(3/7)\*h\*f1(xo,yo\_5)+(2/7)\*h\*f1(xo+h/4,yo\_5+(h/4)\*f1(xo,yo\_5))+(12/9)\*h\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))-(12/7)\*h\*(f1(xo+h/2,yo\_5-(h/2)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))+h\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))))+(8/7)\*h\*(f1(xo+(3/4)\*h,yo\_5+(3/16)\*h\*(f1(xo,yo\_5))+(9/16)\*h\*(f1(xo+h/2,yo\_5-(h/2)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))+h\*(f1(xo+h/4,yo\_5+(h/8)\*(f1(xo,yo\_5))+(h/8)\*(f1(xo+h/4,yo\_5+(h/4)\*(f1(xo,yo\_5))))))))))))))\*h**

**y\_true=f2(xo,yo\_t)**

**yo\_t=y\_true**

**print(xo,'\t',round(yo\_e,2),'\t',round(yo\_h,2),'\t',round(yo\_m,2),'\t',round(yo\_r,2),'\t',round(yo\_3,2),'\t',round(yo\_4,2),'\t',round(yo\_5,2),'\t',round(yo\_t,2))**

**A.append(xo)**

**B.append(yo\_e)**

**C.append(yo\_h)**

**D.append(yo\_m)**

**E.append(yo\_r)**

**F.append(yo\_3)**

**G.append(yo\_4)**

**H.append(yo\_5)**

**I.append(yo\_t)**

**[xo,yo\_e,yo\_h,yo\_m,yo\_r,yo\_3,yo\_4,yo\_5]=[xo+h,yi\_euler,yi\_heun,yi\_mid,yi\_ral,yi\_rk3,yi\_rk4,yi\_rk5]**

**print("\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_")**

**print("x=",A,"\ny(euler)=",[round(i,2) for i in B],"\ny(heun)=",[round(i,2) for i in C],"\ny(mid)=",[round(i,2) for i in D],"\ny(raltson)=",[round(i,2) for i in E],"\ny(3rd order)=",[round(i,2) for i in F],"\ny(4th order)=",[round(i,2) for i in G],"\ny(5th order)=",[round(i,2) for i in H],"\ny(true)=",[round(i,2) for i in I])**

**plt.plot(A,B,label = "Euler\'s Method")**

**plt.plot(A,C,label = "Heun\'s Method")**

**plt.plot(A,D,label = "Midpoint Method")**

**plt.plot(A,E,label = "Ralston's Method")**

**plt.plot(A,F,label = "RK 3rd Order")**

**plt.plot(A,G,label = "RK 4th Order")**

**plt.plot(A,H,label = "RK 5th Order")**

**plt.plot(A,I,'o',label = "True Solution",linestyle = "dashdot")**

**plt.title('RK METHOD (ALL ORDERS)',fontsize=24,)**

**plt.xlabel('x',fontsize=20,)**

**plt.ylabel('f(x,y)',fontsize=20)**

**plt.legend()**

**OUTPUT**

**COMBINE METHOD EXAMPLE**

**xi y-euler y-heun y-mid y-ral y-3rd y-4th y-5th y-true**

**=========================================================================**

**0 1 1 1 1 1 1 1 1.0**

**0.5 5.25 3.44 3.11 3.28 3.22 3.22 3.22 3.22**

**1.0 5.88 3.38 2.81 3.1 3.0 3.0 3.0 3.0**

**1.5 5.12 2.69 1.98 2.35 2.22 2.22 2.22 2.22**

**2.0 4.5 2.5 1.75 2.14 2.0 2.0 2.0 2.0**

**2.5 4.75 3.19 2.48 2.86 2.72 2.72 2.72 2.72**

**3.0 5.88 4.38 3.81 4.12 4.0 4.0 4.0 4.0**

**3.5 7.12 4.94 4.61 4.8 4.72 4.72 4.72 4.72**

**4.0 7.0 3.0 3.0 3.03 3.0 3.0 3.0 3.0**

**=========================================================================**

**x= [0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0]**

**y(euler)= [1, 5.25, 5.88, 5.12, 4.5, 4.75, 5.88, 7.12, 7.0]**

**y(heun)= [1, 3.44, 3.38, 2.69, 2.5, 3.19, 4.38, 4.94, 3.0]**

**y(mid)= [1, 3.11, 2.81, 1.98, 1.75, 2.48, 3.81, 4.61, 3.0]**

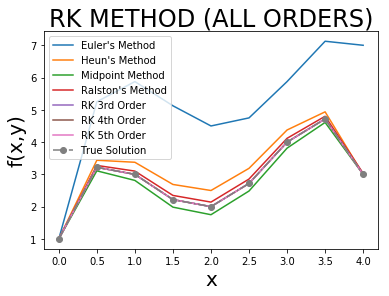
**y(raltson)= [1, 3.28, 3.1, 2.35, 2.14, 2.86, 4.12, 4.8, 3.03]**

**y(3rd order)= [1, 3.22, 3.0, 2.22, 2.0, 2.72, 4.0, 4.72, 3.0]**

**y(4th order)= [1, 3.22, 3.0, 2.22, 2.0, 2.72, 4.0, 4.72, 3.0]**

**y(5th order)= [1, 3.22, 3.0, 2.22, 2.0, 2.72, 4.0, 4.72, 3.0]**

**y(true)= [1.0, 3.22, 3.0, 2.22, 2.0, 2.72, 4.0, 4.72, 3.0]**

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